MATH 462 LECTURE NOTES: LECTURE SEPT 26, 2022

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1. Inner Products

1.1. **Review of analytic geometry.** We reviewed [DFO20, Chapter 3], sections 3.1-3.6

- Definition of norms (normed vector space), 1-norm, 2-norm
- Definition of inner products (inner product space)
- Definition of PSD (symmetric, positive definite) matrix
- Definition of a metric
- Cauchy Schwartz inequality
- Angle between two vectors: $\cos\theta = x^\top y/\|x\|\|y\|$.

We also gave example of the covariance matrix of data (see also [DFO20] Section 10.1). Given $S^m = \{x_1, \dots x_m\}$ with $x_i \in \mathbb{R}^n$.

Definition 1.1. The covariance matrix of S^n is given by

$$C = \frac{1}{m}\sum_{i=1}^{m} x_i x_i^\top$$

Recall that $M = xx^\top$ is the rank 1 $n \times n$ matrix

$$M_{ij} = x_i x_j.$$

References

[DFO20] Marc Peter Deisenroth, A Aldo Faisal, and Cheng Soon Ong. *Mathematics for machine learning*. Cambridge University Press, 2020.

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